

Occupancy Grid Mapping With Uncertain LiDAR Data

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Abstract—High-definition (HD) maps are essential for autonomous vehicles. A promising approach to automated generation of HD maps is occupancy grid mapping using vehicle-borne light detection and ranging (LiDAR) data. Existing occupancy grid mapping methods do not integrate the LiDAR pose uncertainty caused by signal blockage and the multipath effect of global navigation satellite systems (GNSS) into the geometric inverse sensor model (ISM) update. This results in systematic and random errors in the generated maps. To address the problem, this article presents an asymmetric Gaussian Bell ISM that minimizes the systematic and random errors caused by the pose uncertainty in occupancy grid mapping. Our proposed asymmetric Gaussian Bell ISM is evaluated on both a real indoor LiDAR dataset and a small-scale outdoor LiDAR dataset with two designed evaluation metrics. The results demonstrate that the proposed ISM effectively reduces the systematic error in both environments. Specifically, our ISM achieves the lowest systematic error of 7.5 and 5.1 cm for the indoor and outdoor datasets with LiDAR pose uncertainties of 20 and 40 cm, respectively, producing more accurate occupancy grid maps compared to five existing ISMs and indicating its potential for HD map-related applications.

Index Terms—Inverse sensor model (ISM), light detection and ranging (LiDAR), map evaluation, occupancy grid mapping, pose uncertainty.

I. INTRODUCTION

OCCUPANCY grid mapping algorithm is widely employed in mobile robotics to represent the environment as a grid-based map. It decomposes the space into a grid of cells with associated probability values indicating occupancy likelihood [1]. In simultaneous localization and mapping (SLAM), it provides computational efficiency, compatibility with various sensors, and real-time adaptability to changing environments [2], [3]. In autonomous vehicles, the occupancy grid mapping serves as a practical method for generating HD maps, enabling navigation [4], lane-level localization [5], and detailed environment modeling [6]. However, the accuracy of occupancy grid maps is influenced by the LiDAR pose uncertainty, arising from factors such as GNSS errors, motion estimation errors, and

calibration errors, which can lead to misalignments and safety concerns in applications such as autonomous driving [7]. In this work, we focus on the position (translation) component of the LiDAR pose uncertainty, which can be projected onto the beam direction and modeled as range errors in the ISM, and the orientation uncertainty is not considered.

To mitigate the impact of pose uncertainty on the map, researchers often employ techniques such as sensor fusion [8] and use particle filter algorithms [9], [10] to improve pose estimation accuracy. Meanwhile, some researches focus on directly integrating pose uncertainty into the occupancy grid mapping process, such as the Gaussian process (GP) [11], [12], [13] and the fuzzy logic framework [14].

An important component of the occupancy grid mapping algorithm is the inverse sensor model (ISM), which converts each range measurement into per-cell occupancy probabilities along the beam and updates the grid in log-odds (with probability clamping for updatability). Since an exact ISM, which computes the occupancy probability of each grid cell using Bayes' theorem, involves high computational complexity [15], geometric ISMs are widely used as approximations. They employ geometric rules to determine the likelihood that a grid cell is occupied based on sensor readings, such as the Gaussian Bell shape for the measurement uncertainty or the step shape for ideal cases in LiDAR sensors [1]. However, geometric ISMs in previous studies mainly account for small random errors, such as range measurement errors [7], rather than the pose uncertainty, which can be significantly larger and introduces a systematic error in occupancy grid maps.

In this article, we propose a new geometric ISM to incorporate the uncertainty of LiDAR poses into occupancy grid maps by adapting the conventional geometric ISM for measurement uncertainty. Our approach accommodates pose uncertainty directly from a LiDAR sensor during the mapping process, and it distinguishes itself from others by maintaining the simplicity of the occupancy grid framework. The modification is in the process of updating the cell occupancy probabilities rather than the pose estimation.

The main contributions of this article are the following points.

- 1) We demonstrate and quantify the systematic and random errors caused by conventional geometric ISMs under LiDAR pose uncertainty.
- 2) We propose two new evaluation metrics for measuring the random error and systematic error of occupancy grid maps.

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- 3) We propose a new ISM that minimizes the systematic error and quantitatively show its superiority with real indoor and outdoor LiDAR datasets.

Although our experiments are conducted on local-scale indoor and outdoor maps rather than full-scale HD maps, the proposed ISM can be directly integrated into large-scale LiDAR mapping pipelines and HD map generation frameworks, which we leave as future work. Moreover, the orientation component of the pose uncertainty and its impact on maps are not modeled in this work and are also left for future research.

The rest of this article is organized as follows. A review of related works is given in Section II. We then demonstrate the systematic and random errors of existing geometric ISMs under the LiDAR pose uncertainty in Section III, followed by proposing a new ISM and determining the parameters of the proposed ISM in Section IV. Two new evaluation metrics and the 2-D mapping results summary are shown in Section V. Section VI discusses the experiment results, and Section VII concludes this article.

II. RELATED WORK

A. Conventional Occupancy Grid Mapping

Elfes [1] first introduced a probabilistic framework for the occupancy grid mapping methods that decompose the environment into an evenly spaced grid of cells, and each cell in the grid is associated with a probability value to represent the probability of being occupied using the binary Bayesian filter and the Markov assumption. However, early occupancy grid mapping algorithms were limited by the assumptions that cells are independent of each other, and exact poses are known [15]. In this case, the map update is based on geometric ISMs, but, as shown by Thrun [16], such methods cannot accommodate pose uncertainty when mapping with known but uncertain robot poses.

B. Uncertainty Modeling in Occupancy Grid Mapping

Thrun et al. [15] proposed EKF-SLAM, which assumes that there is Gaussian noise in both landmark perception and robot motion to apply an extended Kalman filter to estimate the robot poses and landmark locations, limited by nonlinear motion and observation models. Landmarks are distinctive, recognizable features in the environment that serves as reference points to build a consistent map. Montemerlo et al. [17] further proposed the FastSLAM approach with a Rao-Blackwellized particle filter [18] where each particle represents a hypothesis about the robot's trajectory and map, but FastSLAM suffers from the problem of particle degeneracy and depletion [19].

Based on FastSLAM, Reineking and Clemens [20] developed evidential FastSLAM using Dempster-Shafer theory with its belief function. It quantifies different dimensions of uncertainty caused by contradictory measurements and missing measurements, but it does not quantify the effect of pose uncertainty. In parallel with evidential grid maps, Herrero-Pérez et al. [14] employed fuzzy logic to account for measurement uncertainty and localization uncertainty, but the

aim is to estimate the robot's location instead of mapping the environment with localization uncertainty.

Joubert et al. [7] introduced a method to integrate pose uncertainty directly from an SLAM system into occupancy grid maps. The method utilizes samples from the pose distribution returned by FastSLAM or EKF-SLAM and then adapts the occupancy grid update equation to accommodate these samples using the Monte Carlo approximation. This method benefits occupancy grid maps by incorporating uncertainty from the robot's pose estimation process. It convolves the ideal ISM [1], which assigns probabilities for free-hit-unknown cells along each beam in a step-like model, and a Gaussian centered around the reading to represent the uncertainty in range measurements rather than in sensor poses. The limitation is that it does not quantify the impact of such extra information on the map, which limits its practical applicability.

Another way to model sensor uncertainty is to assume sensor errors to be unknown but constrained within known limits. Langerwisch and Wagner [21] developed an ISM that leverages interval analysis to incorporate measurement and pose uncertainty. However, the mapping outcome is less precise than that of typical probabilistic models and does not account for the sensors' position uncertainty in the model.

C. Non-Deep Learning-Based Occupancy Grid Mapping

Machine learning techniques have been widely applied to occupancy grid mapping to improve adaptability and robustness in complex environments. The independence assumption simplifies the mapping problem over a space as binary estimation problems over cells, but it ignores the spatial correlation between cells stemming from the physical structure of obstacles [22]. To address this limitation, O'Callaghan and Romas [11] employed the GP approach to generate a continuous, nonlinear, and nonparametric function to model occupancy in environments because it can learn spatial dependencies from the dataset. In [23], a method was suggested to incorporate uncertainty in training inputs into a generic GP kernel function, but it is limited by the high computational cost in the training process and has high requirements for the selected samples for learning.

D. Deep Learning-Based Occupancy Grid Mapping

More recently, deep learning-based ISMs have emerged as a new frontier in occupancy grid mapping. Van Kempen et al. [24] developed a simulation-based end-to-end ISM that predicts evidential occupancy grid maps directly from LiDAR point clouds, learning to infer per-cell belief masses and improving occupancy estimates in unobserved but inferable areas. Wirges et al. [25] presented a deep learning-based ISM that augments single-view evidential occupancy grid maps obtained from range sensors such as LiDAR, enabling the network to generate additional evidential structure similar to multiview maps and, thereby, compensate for occlusions and limited viewpoints. Jang et al. [26] employed a deep fusion network that combines LiDAR point clouds and camera images to produce a dynamic occupancy grid map with semantic information, effectively learning a multimodal ISM

that maps fused sensor data to per-cell occupancy, motion, and class labels. Schreiber et al. [27] introduced a multitask recurrent neural network that takes sequences of raw LiDAR data in bird's-eye view representation as input and directly predicts dynamic occupancy grid maps with velocities and semantics, learning an ISM end-to-end and overcoming limitations of geometric ISMs in modeling object shapes and free space.

However, these deep ISMs also come with notable limitations. In almost all of the above works, the ego-vehicle and sensor poses are treated as given by the dataset or simulator, so the networks primarily model measurement noise, occlusions, and semantic ambiguity while leaving map errors induced by pose uncertainty unaddressed [24], [25], [26], [27]. In addition, deep ISMs typically require large, carefully curated training datasets, nontrivial network architectures, and hardware accelerators at run time, and their behavior is difficult to characterize analytically or to relate to the classical probabilistic occupancy grid framework.

Overall, the conventional occupancy grid mapping method using geometric ISMs is renowned for its robustness and ease of implementation, but it is limited by its inability to accommodate pose uncertainty. Technologies such as Gmapping (grid-based FastSLAM) [28], evidential grid maps [29], fuzzy logic mapping [14], and GP occupancy grid maps [30] account for the pose uncertainty, but they either made efforts to improve pose estimation before mapping or sacrificed the simplicity of the traditional occupancy grid framework. Deep neural network-based ISMs show advantages over the geometric ISMs, but, as discussed above, they generally assume accurately known sensor poses and do not explicitly analyze how pose uncertainty distorts the resulting occupancy map.

In summary, relatively little attention has been paid to directly incorporating pose uncertainty into the per-beam geometric ISMs used for cellwise grid updates. Given this knowledge gap, this article addresses the problem of occupancy grid mapping with geometric ISMs under uncertain LiDAR poses and proposes an analytically grounded ISM that explicitly compensates for the pose uncertainty within the classical log-odds framework.

III. MAP ERRORS CAUSED BY LiDAR POSITION UNCERTAINTY

This section demonstrates the performance of existing geometric ISMs under uncertain LiDAR poses in both 1-D mapping and 2-D mapping. We use five ISMs that are widely adopted in occupancy grid mapping, namely, the Octomap ISM [31], the right-truncated Gaussian Bell ISM [32], and the Cartographer ISM [33]. The Octomap ISM and the Cartographer ISM are both ideal ISMs but with different parameters. These five ISMs can provide representative baselines to expose how the pose uncertainty induces map errors in 1-D cases. The cell size is assumed to be 1 cm.

A. Octomap ISM

The Octomap ISM [31] is applied for LiDAR sensors in 3-D occupancy grid mapping. The model defines the occupancy probability of grid cells along the laser beam, assuming

noise-free conditions: an exact sensor pose and no measurement uncertainty. The mathematical definition of the Octomap ISM is shown in the following equation from [31]:

$$P_{occ}(z|Y_{lidar}) = \begin{cases} 0.4, & 0 \leq z < z^* \\ 0.7, & z = z^* \\ 0.5, & z^* < z \leq z_{\max} \end{cases} \quad (1)$$

where z^* is the returned distance and $z_{\max}$ is the maximum perception range of the LiDAR sensor. The occupancy probability values are 0.4 before the returned distance, 0.7 at the returned distance, and 0.5 after the returned distance. This is illustrated in Fig. 1(a).

The Octomap ISM is shown in Fig. 1(a). The dashed line in red represents an obstacle at a distance of 100 cm from the sensor. Fig. 1(f) illustrates how the occupancy probabilities are combined using 1000 LiDAR sensor observations with pose uncertainty. The uncertainty of position is simulated by Gaussian noise with a standard deviation of 5 cm (σ_{sensor}). Each red curve represents one noisy observation. The probabilities are converted to log-odds values during updates, and the accumulated log-odds value of each cell is converted back to probabilities after receiving all measurements. The combined probabilities are limited by a probability clamp from 0.12 to 0.97. As shown in Fig. 1(f), the obstacle represented by the peak of the combined probability is wider than the actual obstacle and is significantly shifted from its actual location.

B. Gaussian Bell ISM

The Gaussian Bell ISM [32] is the truncated convolution of an ideal ISM and a zero-mean Gaussian kernel to model the measurement uncertainty. Equation (2) described in [32] defines the Gaussian Bell ISM

$$P_{occ}(z|Y_{lidar}) = \begin{cases} g(z, z^*), & 0 \leq z \leq z^* \\ \max\{0.5, g(z, z^*)\}, & z^* < z \leq z_{\max} \end{cases} \quad (2)$$

$$g(z, z^*) = (\lambda - \eta) \exp\left(-\frac{(z - z^*)^2}{2\sigma_m^2}\right) + \eta \quad (3)$$

where z^* is the distance returned, λ and η are two scale factors for the function indicating the maximum probability and the minimum probability, respectively, $z_{\max}$ is the maximum range of LiDAR sensor perception, and σ_m^2 is the variance of the measurements. The model will follow the Gaussian function before the returned distance and will be the maximum value between the Gaussian function and 0.5 after the returned distance.

Fig. 1(b) shows the Gaussian Bell ISM where the Gaussian kernel has a standard deviation of 5 cm. The peak probability is 90% decided by λ , and the bottom probability is 10% defined by η . Compared to the step shape in the Octomap ISM, it has a Gaussian Bell shape because it employs a Gaussian model to represent uncertainty in range measurements with smooth changes in probability. The red curves indicate the measurements affected by pose uncertainty, and the blue curve represents the combined probability after log-odds updates under the probability clamp. Fig. 1(g) demonstrates the peak probability shift in the Gaussian Bell ISM. It can be observed that the combined probabilities obtained by the ISM locate the obstacle behind its true position, and its width increases.

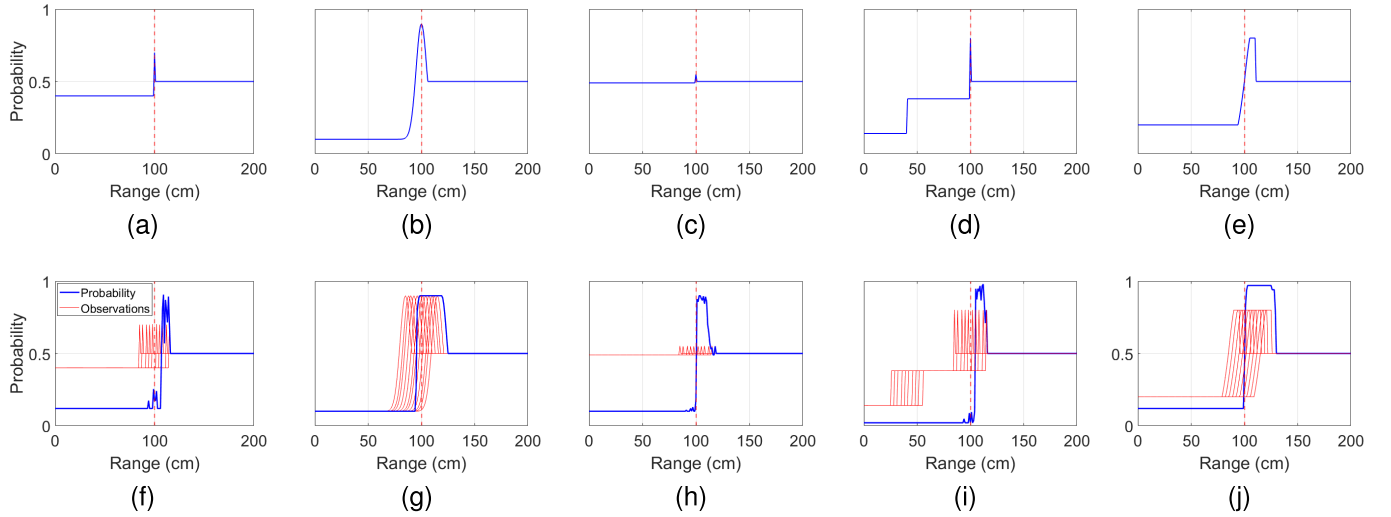


Fig. 1. Peak probability shifts and obstacle width increases of the five existing ISMs with LiDAR position uncertainty ($\sigma_{\text{sensor}} = 5$ cm, cell size = 1 cm, and $z^* = 100$ cm). (a) Octomap ISM (probability clamp [0.12, 0.97]). (b) Gaussian Bell ISM ($\lambda = 0.9$, $\eta = 0.1$, and probability clamp [0.1, 0.9]). (c) Cartographer ISM (probability clamp [0.1, 0.9]). (d) K-NN ISM ($p_{m,\text{in}} = 0.14$, $p_{m,\text{out}} = 0.38$, $p(s) = 0.8$, $s_c = 40$ cm, and probability clamp [0.02, 0.98]). (e) Supereigh2 ISM ($p_{\min} = 0.2$, $p_{\max} = 0.8$, $\sigma(z^*) = 2$ cm, $\tau(z^*) = 10$ cm, and probability clamp [0.12, 0.97]). (f)–(j) Noisy observations in red and the combined probability in blue. Ideally, the highest combined probability should be at the actual position of the obstacle (100 cm), but these five ISMs produce a peak probability shift away from 100 cm and an increase in obstacle width.

C. Cartographer ISM

Cartographer SLAM [34] deals with the accumulation of error estimates of poses by graph optimization [35] instead of particle filters [36]. In the mapping part, Cartographer SLAM applies the Cartographer ISM [33] to update the grid map with a probability clamp to constrain the occupancy probability on the map between 0.1 and 0.9. The following equation shows the Cartographer ISM:

$$P_{\text{occ}}(z|Y_{\text{lidar}}) = \begin{cases} 0.49, & 0 \leq z < z^* \\ 0.55, & z = z^* \\ 0.5, & z^* < z \leq z_{\max} \end{cases} \quad (4)$$

where z^* is the returned distance and $z_{\max}$ is the maximum perception range of the LiDAR sensor. The probability values are 0.49 before the returned distance, 0.55 at the returned distance, and 0.5 after the returned distance.

Fig. 1(c) illustrates the Cartographer ISM that has a shape similar to the Octomap ISM but with different values. Fig. 1(h) shows how the occupancy probabilities are combined given 1000 LiDAR measurements with pose uncertainty simulated by a Gaussian distribution with a standard deviation of 5 cm. The combined probability in blue in Fig. 1(h) shows that the occupied section becomes wider and is behind the true position.

D. K-NN ISM

The K-nearest neighbors (K-NNs) ISM [37] extends the OctoMap ISM by assigning the hit probability according to the local point cloud density around each LiDAR return. For each hit, the average distance s from a point to its K-NN is computed in the point cloud. This allows dense and reliable structures (small s) to contribute higher probabilities near p_u , while sparse or isolated points (large s) are assigned lower

probabilities close to p_l . In addition, the free space in front of the hit is split into an inner and an outer region, with stronger free updates $p_{m,\text{in}}$ near the LiDAR sensor and weaker updates $p_{m,\text{out}}$ farther away.

Let $f(x)$ denote the probability density function of s . The occupancy probability of a point with average distance s is defined as

$$p(s) = p_u - \frac{\int_{-\infty}^s f(x) dx}{\int_{-\infty}^{\infty} f(x) dx} (p_u - p_l) = p_u - F(s) (p_u - p_l) \quad (5)$$

where p_u and p_l are the upper and lower bounds on the occupancy probability, respectively, and $F(s) = \int_{-\infty}^s f(x) dx$ is the cumulative distribution function of the average distance.

Based on this relationship, the K-NN-based ISM can be written as

$$P_{\text{occ}}(z | Y_{\text{lidar}}, s) = \begin{cases} p_{m,\text{in}}, & 0 \leq z < \min\{z^*, s_c\} \\ p_{m,\text{out}}, & s_c \leq z < z^* \\ p(s), & z = z^* \\ 0.5, & z^* < z \leq z_{\max} \end{cases} \quad (6)$$

where z^* is the measured range of the LiDAR return, $z_{\max}$ is the maximum sensor range, and s_c is the maximum inner free-space range along individual beams. The constants $p_{m,\text{in}}$ and $p_{m,\text{out}}$ are the miss probabilities for free-space cells in the inner region ($0 \leq z < \min\{z^*, s_c\}$) and the outer region ($s_c \leq z < z^*$ when $z^* > s_c$), respectively. At the hit location $z = z^*$, the occupancy probability is given by $p(s)$ in (5), where s is the K-NN average distance of the corresponding point. Cells behind the hit remain unknown with probability 0.5.

Fig. 1(d) shows the K-NN ISM that has two different values for the free-space updates. The probability for the inner region

is 0.14 decided by $p_{m,in}$ and 0.38 for the outer region defined by $p_{m,out}$. The distance threshold is 40 cm from s_c , and the hit probability is assumed to be 0.8 from $p(s)$. As shown in Fig. 1(i), after 1000 measurements under the pose uncertainty consistent with previous ISMs, the obstacle indicated by the peak of the combined probability gets wider and is shifted from the true position.

E. Supereight2 ISM

The Supereight2 ISM [38] models measurement uncertainty and a finite wall thickness along each beam. Instead of fixed hit and miss probabilities, Supereight2 ISM uses a piecewise linear function of the distance between a query point and the measured surface. This makes a free-space band in front of the surface, a finite-thickness occupied band behind it, and unknown space elsewhere.

For each LiDAR beam, let z^* be the measured distance of the return and r the distance of a query point along this beam. We define the signed distance

$$d_r = r - z^*$$

which is negative in front of the measured surface ($r < z^*$) and positive behind it ($r > z^*$). The range-dependent uncertainty of the measurement is described by $\sigma(z^*)$, and the wall thickness by $\tau(z^*)$. At the probability level, the model specifies three characteristic values: a strong free-space probability $p_{\min} < 0.5$, a neutral value 0.5, and a strong occupied probability $p_{\max} > 0.5$. Along a single beam, the occupancy probability is then given by

$$P_{occ}(d_r | Y_{lidar}) = \begin{cases} p_{\min}, & d_r \leq -3\sigma(z^*) \\ f_{\text{front}}(d_r), & -3\sigma(z^*) < d_r < 0 \\ f_{\text{back}}(d_r), & 0 \leq d_r < \frac{1}{2}\tau(z^*) \\ p_{\max}, & \frac{1}{2}\tau(z^*) \leq d_r \leq \tau(z^*) \\ 0.5, & d_r > \tau(z^*) \end{cases} \quad (7)$$

$$f_{\text{front}}(d_r) = p_{\min} + \frac{d_r + 3\sigma(z^*)}{3\sigma(z^*)} (0.5 - p_{\min}) \quad (8a)$$

$$f_{\text{back}}(d_r) = 0.5 + \frac{2d_r}{\tau(z^*)} (p_{\max} - 0.5). \quad (8b)$$

Cells far in front of the measured surface are updated toward free space with $p_{\min}$, the probability increases linearly to 0.5 at the surface, then rises from 0.5 to $p_{\max}$ within the first half of the occupied band $\frac{1}{2}\tau(z^*)$, stays at $p_{\max}$ until $\tau(z^*)$, and finally returns to the unknown value 0.5 for points further behind.

To fuse measurements over time, we follow the weighted Bayesian update used in Supereight2 [38]. For each cell M_p , the single-frame log-odds update $l^k(M_p)$ from the ISM is combined with the running mean $\bar{L}^{k-1}(M_p)$ using a capped weight w_k

$$\bar{L}^k(M_p) = \frac{\bar{L}^{k-1}(M_p) w_{k-1} + l^k(M_p)}{w_{k-1} + 1} \\ w_k = \min\{w_{k-1} + 1, w_{\max}\}. \quad (9)$$

The accumulated log-odds is then

$$L^k(M_p) = \bar{L}^k(M_p) w_k \quad (10)$$

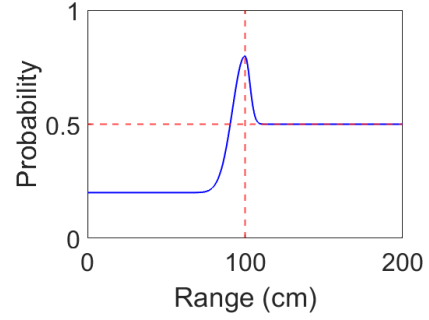


Fig. 2. Proposed asymmetric Gaussian Bell ISM (σ_ℓ , σ_r , λ , and η are assigned arbitrary values; $z^* = 100$ cm).

which keeps an average log-odds while limiting the confidence to at most $w_{\max}$ effective observations.

Fig. 1(e) illustrates the Supereight2 ISM that has a piecewise linear shape when. In Fig. 1(j), the combined probability after 1000 observations shows that the occupied section becomes significantly wider and is located behind the actual position.

IV. PROPOSED ISM

The peak shift problem is caused by the fact that the three existing ISMs assign unequal probability increments to cells on the two sides of the returned distance z^* . When the observations with noise are fused in log-odds, the accumulated contribution has an unbalanced bias around z^* , observed as a peak probability shift. We, therefore, reshape the ISM so that the accumulated contribution is balanced around z^* . The proposed ISM consists of asymmetric left and right lobes defined as

$$w_\ell(z, z^*) = \exp\left(-\frac{(z - z^*)^2}{2\sigma_\ell^2}\right) \times (\lambda - \eta) + \eta \quad (11)$$

$$w_r(z, z^*) = \exp\left(-\frac{(z - z^*)^2}{2\sigma_r^2}\right) \times (\lambda - 0.5) + 0.5 \quad (12)$$

$$P_{occ}(z | Y_{lidar}) = \begin{cases} w_\ell(z, z^*), & 0 \leq z \leq z^* \\ w_r(z, z^*), & z^* < z \leq z_{\max} \end{cases} \quad (13)$$

where z^* is the distance returned, λ is the upper limit of the occupancy probability, η is the lower limit of the occupancy probability, $z_{\max}$ is the maximum range of perception of the LiDAR sensor, σ_ℓ is the standard deviation of the Gaussian function in front of the obstacle, and σ_r is the standard deviation of the Gaussian function behind the obstacle.

The proposed ISM is shown in Fig. 2 and (13). It is an adaptation of the Gaussian Bell ISM in Fig. 1(b), and it is called the asymmetric Gaussian Bell ISM. It allocates η of the occupancy probability to cells in front of the obstacle and λ to the occupied space and a continuous transition to the unknown state behind the obstacle. σ_ℓ and σ_r correspond to the standard deviation of the Gaussian function in front of and behind the obstacle, respectively. The occupancy probability of the cells behind the obstacle remains unknown at 50%.

A. Optimal Values for Parameters in Proposed ISM

Since the values of λ , η , σ_ℓ , and σ_r in the proposed ISM affect the magnitude of the errors in the map, 1-D experiments

have been conducted to determine the optimal values for these four parameters that lead to the minimum map errors. The uncertainty of position σ_{sensor} is represented by the standard deviation on the left side σ_ℓ in our model. Our goal is to choose the ISM parameters so that the peak of the fused probability (or its log-odds) has a stationary maximum at z^* . Therefore, we set the first derivative to zero at z^* .

The proposed ISM is given by (13)

$$P(z) = P_{occ}(z | Y_{lidar}). \quad (14)$$

The log-odds function is defined as

$$\ell(z) = \ln \frac{P(z)}{1 - P(z)}. \quad (15)$$

Its derivative is

$$\ell'(z) = \frac{P'(z)}{P(z)[1 - P(z)]}. \quad (16)$$

With additive Gaussian noise $\epsilon \sim \mathcal{N}(0, \sigma_{sensor}^2)$ as position uncertainty, n independent measurements accumulate log-odds of $z + \epsilon_i$. In the large- N limit, the expected (noise-averaged) curve is

$$S_\ell(z) = \mathbb{E}_\epsilon [\ell(z + \epsilon)] = \int_{-\infty}^{+\infty} \ell(z + u) G(u) du \quad (17)$$

where

$$G(u) = \frac{1}{\sqrt{2\pi} \sigma_{sensor}} e^{-u^2/(2\sigma_{sensor}^2)}. \quad (18)$$

Its derivative is

$$S'_\ell(z) = \int_{-\infty}^{+\infty} \ell'(z + u) G(u) du. \quad (19)$$

The true peak $\hat{z}$ in the presence of noise satisfies $S'_\ell(\hat{z}) = 0$. Writing $\hat{z} = z^* + \Delta$, one has

$$S'_\ell(z^* + \Delta) = 0 \iff \int_{-\infty}^{+\infty} \ell'(z^* + \Delta + u) G(u) du = 0. \quad (20)$$

Expanding $S'_\ell(z^* + \Delta)$ in powers of Δ up to second order

$$S'_\ell(z^* + \Delta) = S'_\ell(z^*) + S''_\ell(z^*) \Delta + \frac{1}{2} S'''_\ell(z^*) \Delta^2 + \mathcal{O}(\Delta^3). \quad (21)$$

We define the coefficients at $z = z^*$

$$a = \frac{1}{2} S'''_\ell(z^*), \quad b = S''_\ell(z^*), \quad c = S'_\ell(z^*). \quad (22)$$

Then, the stationary condition $S'_\ell(z^* + \Delta) = 0$ becomes the quadratic equation

$$a \Delta^2 + b \Delta + c = 0. \quad (23)$$

Provided $a \neq 0$, the solutions are

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad (24)$$

An unbiased estimate at $\Delta = 0$ requires this to be a double root, i.e., $c = 0$ in (23). Hence, the unbiased-peak condition is

$$c = \frac{\lambda - \eta}{\sigma_\ell^2 \lambda (1 - \lambda)} - \frac{\lambda - 0.5}{\sigma_r^2 (0.25)} = 0. \quad (25)$$

Rearranging (25) gives the desired relation between the left- and right-width parameters

$$\sigma_r^2 = \frac{4(\lambda - 0.5)\lambda(1 - \lambda)}{\lambda - \eta} \sigma_\ell^2. \quad (26)$$

When σ_ℓ and σ_r satisfy (26), the root $\Delta = 0$ is a double root of (23), so the model exhibits zero systematic error in its peak probability estimate for any level of position uncertainty. Since λ is in $(0.5, 1)$ and η is in $(0, 0.5)$, σ_ℓ must be greater than σ_r , which means that the left part of the model is always wider than the right part.

B. Quantification of Map Errors

Two evaluation metrics are proposed to quantify the systematic error that appears in the map as a shift in the peak probability (or simply displacement of occupied surfaces), and the random error that appears in the map as thick occupied surfaces.

Mean Signed Distance: Defined as the average signed distance from all occupied cells (probability greater than 0.65) on the shifted and thickened map to the reference map, quantifying the peak probability shift.

Let $\Omega = \{i | P_{shift}(i) > 0.65\}$ be the set of occupied cells on the shifted and thickened map, and let $N = |\Omega|$. For each cell $i \in \Omega$, define its signed distance

$$d_i = s_i \min_{j \in \Omega_{ref}} \|\mathbf{x}_i - \mathbf{x}_j\|_2 \quad (27)$$

$$s_i = \begin{cases} -1, & \text{obstacle moves toward the trajectory} \\ +1, & \text{obstacle moves away from the trajectory} \end{cases} \quad (28)$$

where Ω_{ref} is the occupied set in the reference map.

Then, the MSD is

$$\text{MSD} = \frac{1}{N} \sum_{i \in \Omega} d_i. \quad (29)$$

A larger MSD reflects a stronger systematic shift of the obstacles (inward if negative and outward if positive).

Mean Absolute Deviation: To isolate the random error, the shift captured by MSD is subtracted from each signed distance, and the residual dispersion quantifies obstacle thickness. Define the residual $r_i = d_i - \text{MSD}$. The mean absolute deviation (MAD) is then

$$\text{MAD} = \frac{1}{N} \sum_{i=1}^N |r_i| = \frac{1}{N} \sum_{i=1}^N |d_i - \text{MSD}|. \quad (30)$$

A larger MAD indicates stronger random dispersion and thicker obstacles, whereas a value close to zero implies very little scatter and narrow obstacles.

This article focuses on MSD as the more significant of the two, as it results in the incorrect location of obstacles in the map.

Once the relation between σ_ℓ and σ_r is enforced, λ and η can be selected by inspecting their resulting performance of MSD and MAD for the proposed ISM in Fig. 2 in the 1-D simulation experiments.

Fig. 3 presents contour maps of MSD and MAD as functions of (λ, η) . Each pair of (λ, η) on the cyan curve in Fig. 3(a) corresponds to zero MSD. This curve is obtained numerically by sampling λ and, for each value, searching for η such that MSD comes to 0. Among these candidates, $(0.941, 0.136)$ is chosen because it simultaneously shows a relatively low MAD in Fig. 3(b) and avoids extreme log-odds values during map

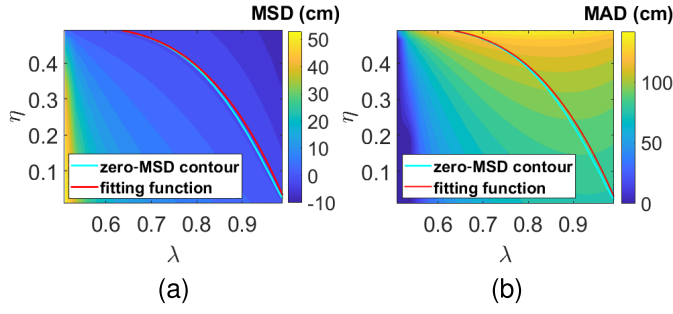


Fig. 3. MSD and MAD contour maps for different λ 's and η 's ($\sigma_{\text{sensor}} = 20$ cm and $z^* = 100$ cm). (a) MSD contour map. (b) MAD contour map.

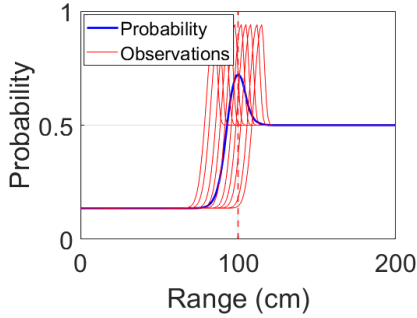


Fig. 4. Peak probability shifts and obstacle width increases of the proposed ISM with LiDAR position uncertainty ($\sigma_\ell = \sigma_{\text{sensor}} = 5$ cm, $\sigma_r = 1.743$ cm, $\lambda = 0.941$, $\eta = 0.136$, cell size = 1 cm, and $z^* = 100$ cm).

updates when probabilities approach 0 or 1. In all subsequent indoor and outdoor experiments, this parameter pair is used without further tuning. The cubic function in (31) is introduced as a compact analytic approximation of the zero-MSD contour line. It allows users to interpolate admissible (λ, η) pairs along this curve without rerunning the full MSD/MAD grid search

$$\eta(\lambda) = 1 - (4\lambda^3 - 6\lambda^2 + 3\lambda). \quad (31)$$

C. Peak Probability Shift for Proposed ISM

Similar to Fig. 1(f)–(j), Fig. 4 shows the systematic and random errors of the proposed ISM in 1-D. Given σ_{sensor} at 5 cm, the values of σ_ℓ , σ_r , λ , and η for the proposed ISM could be confirmed according to (26) and (31). It can be observed that the highest combined probability is exactly at the actual position of the obstacle (100 cm) without systematic error, validating the effectiveness of the proposed ISM.

V. EXPERIMENTS

A. 2-D Maps' Evaluation

The 2-D maps for indoor environments in Fig. 5 are based on the actual odometry and LiDAR measurements offered by the University of Pennsylvania [39]. Fig. 5(a) shows the reference map with exact LiDAR poses using the Octomap ISM, and the shape and position of obstacles in white on the reference map are clear and precise. Fig. 5(b)–(f) shows the maps with the LiDAR position uncertainty using the Octomap ISM, the Gaussian Bell ISM, the Cartographer ISM, the K-NN ISM, and the Supereight2 ISM, respectively. As shown

TABLE I

QUANTITATIVE EVALUATION OF MAP ERROR ON THE INDOOR DATASET

ISMs	MSD (cm)	MAD (cm)
Octomap (Fig. 5b)	19.2	19.7
Gaussian Bell (Fig. 5c)	31.1	31.3
Cartographer (Fig. 5d)	16.6	18.2
K-NN (Fig. 5e)	12.8	19.0
Supereight2 (Fig. 5f)	26.4	19.8
Proposed (Fig. 5g)	7.5	10.4

TABLE II

QUANTITATIVE EVALUATION OF MAP ERROR ON THE OUTDOOR DATASET

ISMs	MSD (cm)	MAD (cm)
Octomap (Fig. 6b)	11.9	24.8
Gaussian Bell (Fig. 6c)	13.4	34.3
Cartographer (Fig. 6d)	8.4	21.4
K-NN (Fig. 6e)	14.2	26.5
Supereight2 (Fig. 6f)	20.9	29.0
Proposed (Fig. 6g)	5.1	25.3

by the red trajectories on the maps, the pose uncertainty is simulated by noise following a zero-mean Gaussian distribution with a standard deviation of 20 cm. The noise level ($\sigma_{\text{sensor}} = 20$ cm) corresponds to two pixels in the 2-D maps (pixel size 10 cm), providing a moderate and visible pose uncertainty for analysis. Compared to the reference map, the obstacles (particularly the walls) in Figs. 5(b)–(f) shift away from the LiDAR sensor trajectory demonstrated by wider empty spaces and their thickness increases, which is aligned with the peak probability shift and width increase in Fig. 1(f)–(j). Fig. 5(g) shows the map generated by the proposed ISM. It shows a weaker peak probability shift, and the obstacles are thinner compared to Fig. 5(b)–(f).

As shown in Table I, the systematic error (MSD) and the random error (MAD) for each map in Fig. 5 are listed, omitting the reference map Fig. 5(a). Since Fig. 5(a) is considered the reference map in the evaluation, it has zero systematic error and random error compared to itself. Fig. 5(c) shows the highest MSD and MAD, which corresponds to its most obvious shift and the thickest obstacles in the visual check. In contrast, Fig. 5(g) has the best performance in both MSD and MAD, followed by Fig. 5(d) and (e). Overall, Table I demonstrates that the proposed ISM has the best performance on both MAD and MSD compared to other existing ISMs when σ_{sensor} reaches 20 cm.

The 2-D outdoor mapping results shown in Fig. 6 are based on the KITTI Odometry dataset [40]. Since KITTI provides the ground-truth poses for the first 11 sequences, we choose a section of sequence 05, which was collected in built environments, as another data source to validate the generalization and practical deployability of our method. The 2-D point clouds are extracted as planar LiDAR scans from the 3-D point clouds.

Fig. 6(a) shows the reference map with ground-truth poses using the Octomap ISM, showing the thin and clear obstacle surfaces along the road. The noise level ($\sigma_{\text{sensor}} = 40$ cm) corresponds to four pixels in the maps (pixel size 10 cm). Compared to the reference map, the obstacles in Fig. 6(f) shift away from their actual position by 2 pixels on average, as

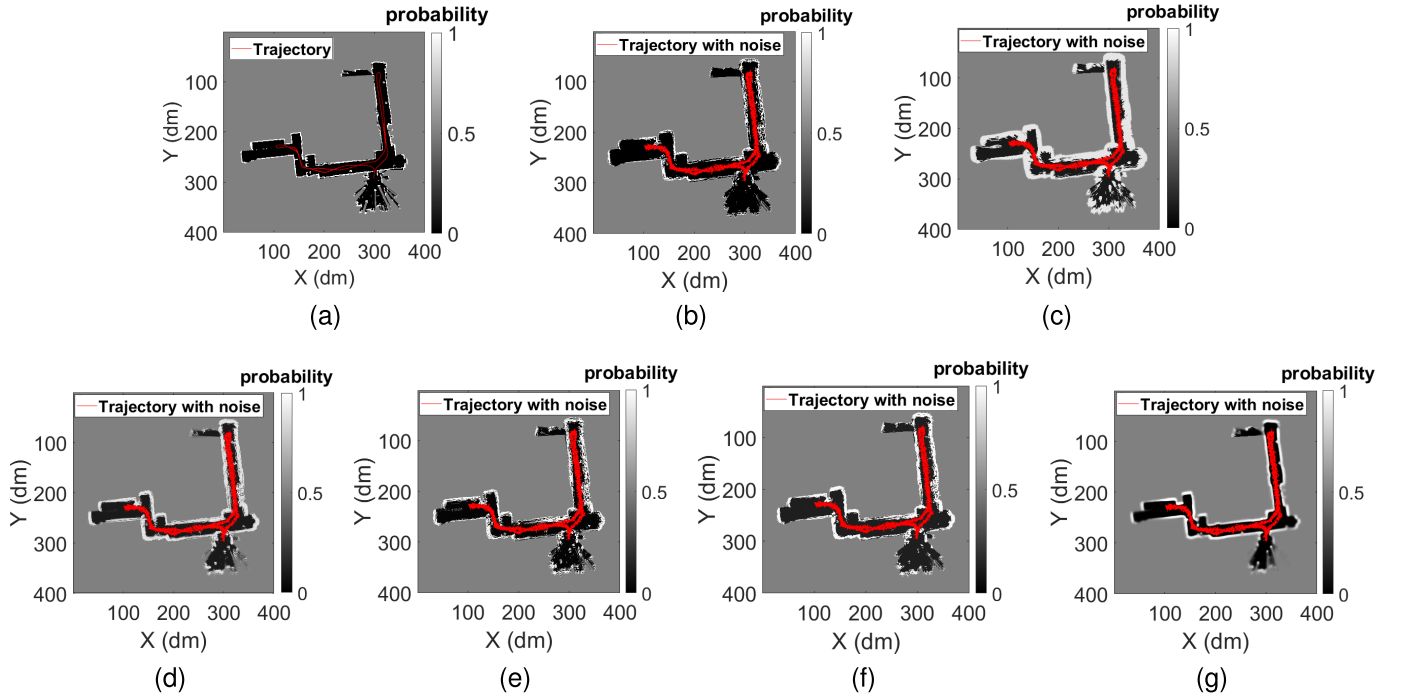


Fig. 5. 2-D maps based on the dataset from the University of Pennsylvania using different ISMs under LiDAR position uncertainty ($\sigma_{\text{sensor}} = 20$ cm and pixel size = 10 cm). (a) Reference map generated by the Octomap ISM with no trajectory noise exhibits no probability peak shift and thin obstacles. (b) Map generated by the Octomap ISM with the noisy trajectory shows a significant peak shift (note the wider empty spaces) and obstacle thickening. (c) Map generated by the Gaussian Bell ISM with the noisy trajectory exhibits a strong peak shift and obstacle thickening. (d) Map generated by the Cartographer ISM with the noisy trajectory exhibits a strong peak shift and obstacle thickening. (e) Map generated by the K-NN ISM with the noisy trajectory exhibits a weak peak shift and obstacle thickening. (f) Map generated by the Supereight2 ISM with the noisy trajectory exhibits a strong peak shift and obstacle thickening. (g) Map generated by the proposed ISM with the noisy trajectory exhibits minimal peak shift and obstacle thickening.

shown by the MSD in Table II. Fig. 6(c) still shows the largest obstacle thickness by the visual check, which is consistent with its MAD in Table II. Fig. 6(g), created by the proposed ISM, shows an invisible peak probability shift of less than one pixel, and the obstacles' thicknesses are in the middle compared to other ISMs. The MAD of the proposed ISM is only slightly higher than the best result in Table II.

B. Comparison of 2-D Map Errors

The six ISMs in (1), (2), (4), (6), (7), and (13) are applied to the mapping process at different levels of LiDAR position uncertainty.

For the indoor environment, Fig. 7 shows the results for various levels of position uncertainty when the cell size is 10 cm, and Fig. 8 shows results when the cell size is 5 cm. It can be observed that the proposed ISM performs the best on both systematic error (MSD) and random error (MAD) when the LiDAR position uncertainty is greater than 5 cm, compared to the other ISMs, in both cell sizes.

Fig. 9 shows the results for the outdoor environment when the map cell size is 10 cm. It demonstrates that the proposed ISM has the lowest systematic error when the LiDAR position uncertainty gets large (greater than 10 cm) and the moderate random error among these six ISMs.

C. Runtime Comparison

The experiments were conducted on a laptop equipped with an Intel Core i5-13500H processor and 16 GB of RAM,

TABLE III
RUNTIME COMPARISON FOR ISMS

ISMs	Indoor (ms per scan)	Outdoor (ms per scan)
Octomap	7.632	7.049
Gaussian Bell	14.362	14.298
Cartographer	7.475	7.189
K-NN	11.330	10.577
Supereight2	17.677	15.146
Proposed ISM	12.803	13.840

running Windows 11 (24H2) and MATLAB R2024a. Table III reports the runtime per scan for the mapping pipeline on the indoor and outdoor datasets in Figs. 5 and 6.

The Octomap ISM and the Cartographer ISM are faster than the other ISMs because they both use step-like ideal ISMs with different probability values. In contrast, the runtime of the Gaussian Bell ISM is substantially longer. This is due to the Gaussian Bell-shaped kernels used in both the Gaussian Bell and the proposed ISMs, which are more computationally demanding than the step-like models. The Supereight2 ISM is the slowest, as its distance-dependent piecewise linear model has the highest computational load.

The proposed asymmetric Gaussian Bell ISM lies between the step-like and the Supereight2 models in terms of runtime and remains lightweight in practice. In our implementation, its runtime per scan is well below 20 ms on both datasets (see Table III). Therefore, for a LiDAR sensor with a scan period of 20 ms (50 Hz), the proposed ISM is sufficiently efficient for real-time occupancy grid mapping.

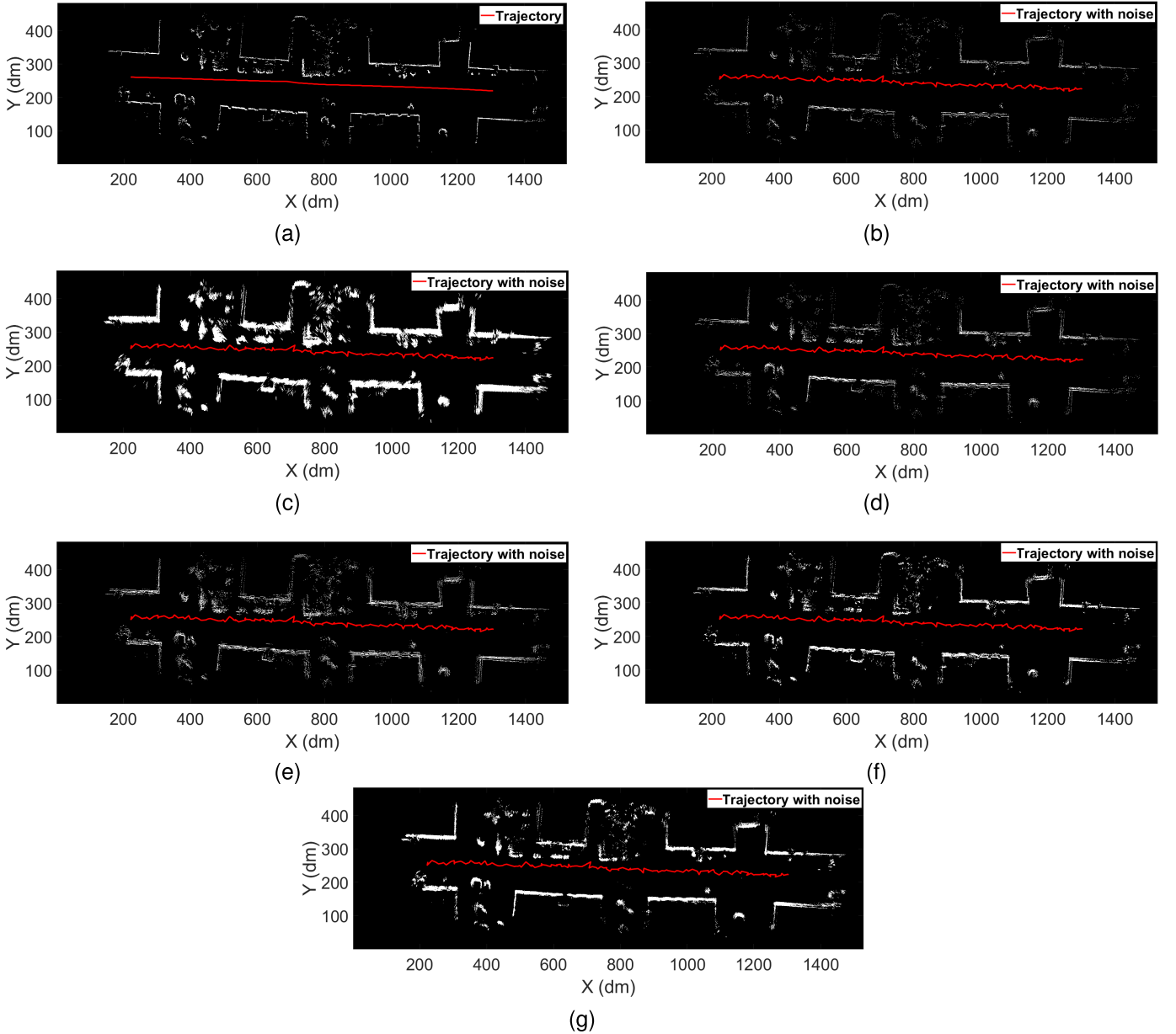


Fig. 6. 2-D maps based on the KITTI Odometry dataset using different ISMs under LiDAR position uncertainty ($\sigma_{sensor} = 40$ cm and pixel size = 10 cm). (a) Reference map created by the Octomap ISM with ground-truth poses. (b) Map created by the Octomap ISM with the noisy trajectory. (c) Map created by the Gaussian Bell ISM with the noisy trajectory. (d) Map created by the Cartographer ISM with the noisy trajectory. (e) Map created by the K-NN ISM with the noisy trajectory. (f) Map created by the Supereight2 ISM with the noisy trajectory. (g) Map created by the proposed ISM with the noisy trajectory.

VI. DISCUSSION

A. Difference Between 1-D and 2-D Experimental Results

According to Fig. 3 in the 1-D experiments, the proposed ISM is supposed to generate a map with zero systematic error (MSD), but the proposed ISM in Figs. 7–9 shows nonzero MSD that increases as σ_{sensor} increases. There are two reasons. One is the limited number of measurements caused by the datasets that cannot satisfy the large- N limit for (17). Another reason is that the high-order terms are removed in (21) in the Taylor expansion, and the truncation error caused by the high-order terms is not negligible when σ_{sensor} gets relatively high values. As shown in Figs. 7–9, the truncation error could be ignored when σ_{sensor} is less than 10 cm because the resulting

MSD is usually less than half of the cell size and could be ignored on the map.

B. Generalization to Large-Scale Maps

In large-scale applications such as HD maps for autonomous driving, the extent of the map will be much larger, and the cell size should also increase proportionally to enable managing the massive size of the map. The LiDAR pose error σ_{sensor} would also increase due to the larger pose estimation errors in large outdoor environments. The effect of the proportion of the cell size to σ_{sensor} on map quality can be explored based on Figs. 7–9. It is found that when σ_{sensor} is less than twice the cell size, the systematic error (MSD) often gets smaller

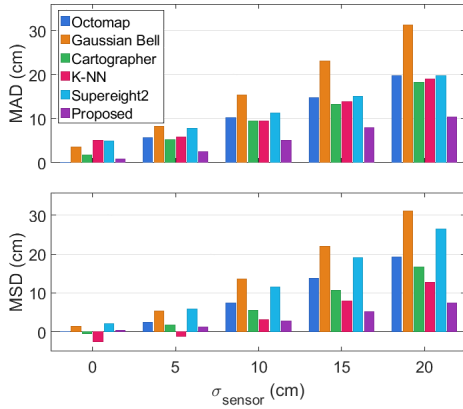


Fig. 7. Comparison of map errors for different ISMs evaluated on the indoor dataset (pixel size = 10 cm).

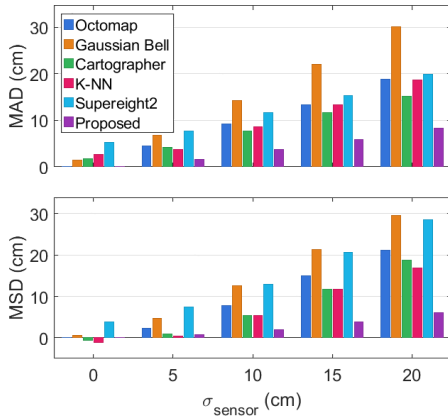


Fig. 8. Comparison of map errors for different ISMs evaluated on the indoor dataset (pixel size = 5 cm).

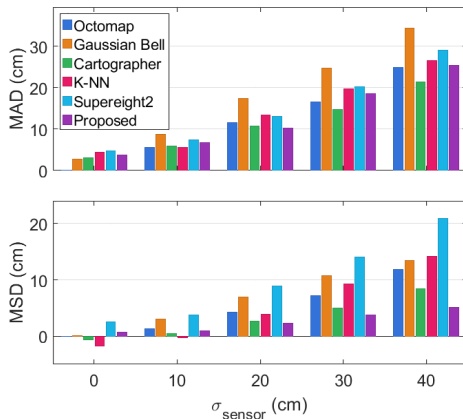


Fig. 9. Comparison of map errors for different ISMs evaluated on the outdoor dataset (pixel size = 10 cm).

than the cell size. The experiments are conducted based on the datasets of an indoor environment and a small-scale outdoor environment, but such a proportion might be helpful when considering the cell size in outdoor mapping for large-scale environments.

VII. CONCLUSION

In this work, we introduced an asymmetric Gaussian Bell geometric ISM that explicitly accounts for the LiDAR position uncertainty in occupancy grid mapping. Existing geometric ISMs suffer from two types of map errors: the obstacle peak shift (systematic error) and the obstacle thickening (random error) under the LiDAR position uncertainty. Across a real indoor and outdoor LiDAR dataset on multiple uncertainty levels, our ISM consistently reduces both MSD and MAD, outperforming the Octomap, Gaussian Bell, Cartographer, K-NN, and Supereight2 ISMs.

Our method creates more accurate maps that have less obstacle shift and reduced obstacle thickening, which is beneficial to both localization and planning. We introduce two new evaluation metrics, MSD and MAD, which provide a practical basis for evaluating ISMs under uncertain LiDAR poses. For application, the proposed ISM could be used in 2-D occupancy grid mapping. The ISM parameters are based on σ_{sensor} and the resolution of the map, and the obstacles need to be thinned to their centerlines.

The pose of a LiDAR sensor consists of position and orientation, but this article only models the position uncertainty, and the orientation uncertainty is ignored. Future work will incorporate orientation uncertainty within the same geometric framework and extend the method to 3-D occupancy grid mapping.

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